APPENDIX A: WANSKIAN RECURRENCE RELATIONS

$$zI'_v(z)+vI_v(z) = zI_{v-1}(z),$$
$$zI'_v(z)-vI_v(z) = zI_{v+1}(z)$$
$$zK'_v(z)+vK_v(z) = -zK_{v-1}(z),$$
$$zK'_v(z)-vK_v(z) = -zK_{v+1}(z)$$
$$zJ'_v(z)+vJ_v(z) = zJ_{v-1}(z),$$
$$zJ'_v(z)-vJ_v(z) = -zJ_{v+1}(z)$$
$$I'_0(z) = I_1(z), \quad K'_0(z) = -K_1(z)$$
$$J'_0(z) = -J_1(z), \quad Y'_0(z) = -Y_1(z)$$
$$J_v(z)Y'_v(z)-Y_v(z)J'_v(z) = \frac{2}{\pi z}$$
$$I_v(z)K'_v(z)-K_v(z)I'_v(z) = -\frac{1}{z}$$
$$I_v(z)K_{v+1}(z)+K_v(z)I_{v+1}(z) = \frac{1}{z}$$
$$J_v(z\,e^{m\pi i}) = e^{mv\pi i}J_v(z),$$
$$I_v(z\,e^{\pm 1/2\pi i}) = e^{\pm 1/2v\pi i}J_v(z)$$
$$K_v(z\,e^{\pm 1/2\pi i}) = \pm\tfrac{1}{2}\pi i\,e^{\mp 1/2v\pi i}[-J_v(z)\pm iY_v(z)]$$
$$Y_v(z\,e^{m\pi i}) = e^{-mv\pi i}Y_v(z)+2i\sin(mv\pi)\cot(v\pi)J_v(z)$$

APPENDIX B: EXAMPLES TO CALCULATE ROOTS β_n OF EQUATION (17)

$$F_1(\beta) = \frac{Y_0(\beta r_i)J_1(\beta r_0)-J_0(\beta r_i)Y_1(\beta r_0)}{Y_1(\beta r_i)J_1(\beta r_0)-J_1(\beta r_i)Y_1(\beta r_0)}$$
$$F_2(\beta) = \frac{\lambda_w}{ah_w}\left(\frac{1}{\tau_f\beta}-a\beta\right)$$

If $F_1(\beta) = F_2(\beta)$, then $\beta = \beta_n$.

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Heat wave phenomena in IC chips

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INTRODUCTION

Thermal management is becoming a predominant consideration in the design of IC chips and their packaging. The electrical behavior of devices and their reliability are strongly dependent both on the temperature of chip and temperature difference among the components. Many researchers pay their attention to the failure resulting from an overhigh chip temperature, which is always associated with irreversible mechanical fracture as well as loss of electrical functions. In contrast our efforts have concentrated on the analysis of the thermal failure arising from temperature difference among the components related to critical electrical paths. For a high-speed system, the component's performance is sensitive to the temperature difference between them because of the problem of signal skew, and so, the junction temperatures of various components should be kept within a specified range for a high performance system. For most chips, 0.25°C may be the maximum allowable value.

When a thermal analysis was applied to the chip at the component level, the following Fourier's heat conduction equation was usually used

$$q = -K\frac{\partial T}{\partial x}. \quad (1)$$

However this equation is based on the diffusion mechanism and implies a presumption of infinite thermal propagation speed not applicable for a rapid wave heat transient process. Alternatively, the C–V heat conduction equation, originally proposed from Maxwell equation [1] and then modified by Cattaneo and Vernotte [2–4], can be used for the description of such a rapid heat conduction

$$\tau\frac{\partial q}{\partial t}+q = -K\frac{\partial T}{\partial x} \quad (2)$$

where τ, defined as $\tau = \alpha/c^2$, is the relaxation time and explained as the build-up period of the commencement of heat flow after a temperature gradient is imposed on the medium; c, the thermal wave propagation speed and α, the thermal diffusivity of the medium.

Comparing the C–V equation (2) with the telegram equation

$$\frac{L}{R}\frac{\partial i}{\partial t}+i = -\frac{1}{R}\frac{\partial E}{\partial x} \quad (3)$$

we have the complete analogy between heat transfer and electrical current transmission as listed in Table 1. In electrical analogy, the term τ/K in equation (2) is equivalent to the electrical inductance L. The relaxation term can not be neglected in strongly transient process as that the electrical inductance can not be ignored for an rapid alternating current circuit. This is known as thermal wave phenomena.

Several features of high speed IC chip make it important

NOMENCLATURE

c	velocity of thermal wave propagation [m s^{-1}]	t	time [s]
C_p	thermal capacity [J kg^{-1} K^{-1}]	T	temperature [K]
K	thermal conductivity [W m^{-1} K^{-1}]	x	dimensionless radius.
g	volumetric heat generation [W m^{-3}]		
G	dimensionless heat generation		
h	dimensionless time		
q	heat flux [W m^{-2}]		
r	radius [m]		

Greek symbols

α	thermal diffusivity [m^2 s^{-1}]
ρ	density [kg m^{-3}]
θ	dimensionless temperature
τ	thermal relaxation time [s].

Table 1. Electrical analogy of heat conduction

Parameters	Flow	Potential	Resistance	Capacity	Inductance
Electrical	i	E	R	C	L
Thermal	q	T	$1/K$	ρC_p	τ/K

to investigate thermal wave phenomena. The first one is the high frequency of electrical pulse, which may be as high as 1 GHz to 1 THz in current IC. Consequently the IC elements would undergo a very rapid transient process. The relaxation term in C–V equation (2) becomes important in this situation because $\tau\,\partial q/\partial t$ is of great value. The second feature is the high purity of the IC base materials. For single crystal silicon used as IC base, the admixture is less than 0.1 ppb. In such a medium with perfect lattice structure, heat transferred by phonons would have a great contribution to heat conduction. Then the heat conduction would behave as a wave propagation. Moreover, it is believed that the possible cryogenic operating conditions introduced to speed up the system will enhance the wave phenomena. In fact one of the earliest experiments showing the wave behavior of temperature was performed in superfluid liquid helium by Peshkov.

This paper addresses heat conduction in IC chip with special attention paid to the evaluation of heat wave effect and its importance for thermal analysis and design of IC chip.

ANALYSIS

As a simple model with emphasis on the evaluation of heat wave effects, a general IC element, for example, a resistor or a p–n junction, prepared with special technique such as doping with foreign substance onto the silicon base, was simply considered as a semi-spherical region (Fig. 1) with thermal physical properties such as those of silicon. Oscillatory current passes through and as a result Joule heat is generated inside this small region. If the surface is regarded as thermally insulated and the reflection of thermal wave on it was ignored, the problem can be taken as a one-dimensional (1D) problem of heat conduction in an infinite solid, because the size of heating region (in the order of μm) is very small compared with the depth of silicon base (in the order of mm).

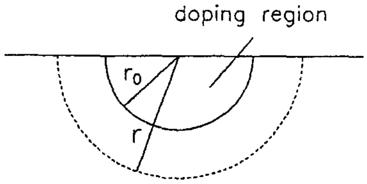


Fig. 1. Simplified thermal model of IC element in silicon base.

Combining the C–V equation (2) with the energy equation in the spherical coordinate system results in the following hyperbolic heat conduction equation

$$\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r}=\frac{1}{\alpha}\left(\tau\frac{\partial^2 T}{\partial t^2}+\frac{\partial T}{\partial t}\right)-\frac{1}{K}\left(\tau\frac{\partial g}{\partial t}+g\right) \quad (4)$$

where $g(r,t)$ represents the volumetric energy source in the medium. The boundary conditions and initial conditions are respectively

$$\left.\frac{\partial T}{\partial r}\right|_{r=0}=0 \quad \text{and} \quad T|_{r\to\infty}=0 \quad (5)$$

$$T|_{t=0}=0 \quad \text{and} \quad \left.\frac{\partial T}{\partial t}\right|_{t=0}=0. \quad (6)$$

The following dimensionless quantities are introduced for mathematical convenience in the analysis

$$x=\frac{r}{2\sqrt{(\alpha\tau)}} \qquad h=\frac{t}{2\tau}$$

$$\theta=\frac{\rho C_p}{4g_o\tau}\frac{x}{x_o}T \qquad G=\frac{x}{x_o}\frac{g}{g_o}$$

where constant term g_o represents the maximum value of volumetric energy generation in the whole medium and duration. Applying the Fourier sine integration transforms pair in space variables as:

$$\bar{\theta}(\beta,h)=\int_0^\infty \theta(x,h)\sin(\beta x)\,\mathrm{d}x \quad (7)$$

$$\theta(x,h)=\frac{2}{\pi}\int_0^\infty \bar{\theta}(\beta,h)\sin(\beta x)\,\mathrm{d}x \quad (8)$$

we transfer the partial differential equations (4)–(6) to simple differential equations:

$$\frac{\mathrm{d}^2\bar{\theta}}{\mathrm{d}h^2}+2\frac{\mathrm{d}\bar{\theta}}{\mathrm{d}h}+\beta^2\bar{\theta}=\frac{1}{2}\frac{\mathrm{d}\bar{G}}{\mathrm{d}h}+\bar{G} \quad (9)$$

$$\bar{\theta}|_{h=0}=0 \quad \text{and} \quad \left.\frac{\mathrm{d}\bar{\theta}}{\mathrm{d}h}\right|_{h=0}=0. \quad (10)$$

The solution of equations (9) and (10) can be obtained by a rather lengthy but straightforward series of manipulations including careful substitution of conditions and the inverse transformation. After arrangement, the solution appears as

$$\theta(x,h) = \frac{2}{\pi}\int_{\beta=0}^{\infty}\int_{h'=0}^{h}\int_{x'=0}^{\infty} \times\left(G(x',h')+\frac{1}{2}\frac{\mathrm{d}G(x',h')}{\mathrm{d}h'}\right)\times\sin(\beta x')\,\mathrm{e}^{-(h-h')} \times\frac{\sin(\sqrt{(\beta^2-1)}(h-h'))}{\sqrt{(\beta^2-1)}} \times\sin(\beta x)\,\mathrm{d}x'\,\mathrm{d}h'\,\mathrm{d}\beta. \quad (11)$$

The energy generation $g(r, t)$ of single pulse in IC element is assumed to be a square-corner switching wave form energy confined in the region of $r < r_0$,

$$G = \begin{cases} \dfrac{x'}{x_o} & \text{for } 0 < x' < x_o \text{ and } 0 < h' < h_o \\ 0 & \text{else} \end{cases} \quad (12)$$

where x_o and h_o are dimensionless distance and dimensionless time corresponding to doped sphere radius r_o and heating pulse time t_o.

Assuming the order of integration of equation (11) is interchangeable, paying attention to the term of $\mathrm{d}G(x', h')/\mathrm{d}h'$, and introducing new variables $m = x-x'$, $n = h-h'$, the final solution for the problem will be

$$\theta = \int_{n=h_1}^{h}\int_{m=x-x_o}^{x+x_o}\frac{x-m}{2x_o} \cdot\mathrm{e}^{-n}\cdot\left(I_o(\sqrt{(n^2-m^2)})+\frac{n}{\sqrt{(n^2-m^2)}}I_1(\sqrt{(n^2-m^2)})\right)\mathrm{d}m\,\mathrm{d}n \quad \text{for } n > |m| \quad (13)$$

where

$$h_1 = \begin{cases} h-h_o & \text{for } h > h_o \\ 0 & \text{for } h < h_o \end{cases} \quad (14)$$

I_0 and I_1 are modified Bessel functions of the first kind of zero order and first order, respectively.

The temperature profile both in the doped area and the silicon base along all the duration can be determined from the numerical integration of equation (13) or a straightforward but tedious integration of the series term, if Bessel function is expressed as series term.

RESULTS AND DISCUSSIONS

Figure 2 shows a typical plot of temperature vs position at various times obtained from equation (13) with a rather large relaxation time of 10^{-5} s. The striking feature of the graph is the wave behavior of temperature, i.e. an energy pulse in a small region gives rise to a thermal wave front during a short time period which travels in the medium with a finite velocity of about 2.5 m s^{-1}. Unlike the results of diffusion equation, the temperature rise obtained from wave equation is limited within a small region and the region beyond remains undisturbed. The heat conduction acts as the transport of the 'energy bulk' with higher temperature.

The wave nature of temperature in Fig. 2 is similar to that observed in liquid helium by Peshkov and examined theoretically by Vick and Ozisik [5], when an energy source with a wave form of delta function was applied, i.e. total energy is released spontaneously at time $t = 0$, in spite of the fact that the wave shape is not as sharp as that given by Vick and Ozisik.

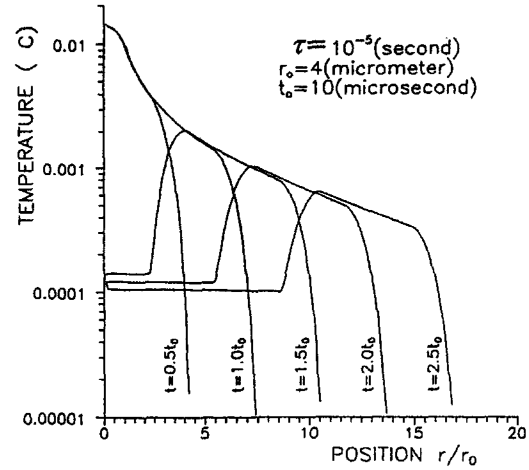


Fig. 2. Heat wave propagation in the medium with a finite velocity of 2.5 m s^{-1}.

Figure 3 illustrates a temperature profile in medium at time $t = 2\times t_0$ with different relaxation time. It is clear that the wave behavior of temperature creates the higher peak temperature than that predicted by pure diffusion mechanism of heat transport. With the decrease of relaxation time τ, the heat transport dominated by wave switches gradually to that dominated by diffusion.

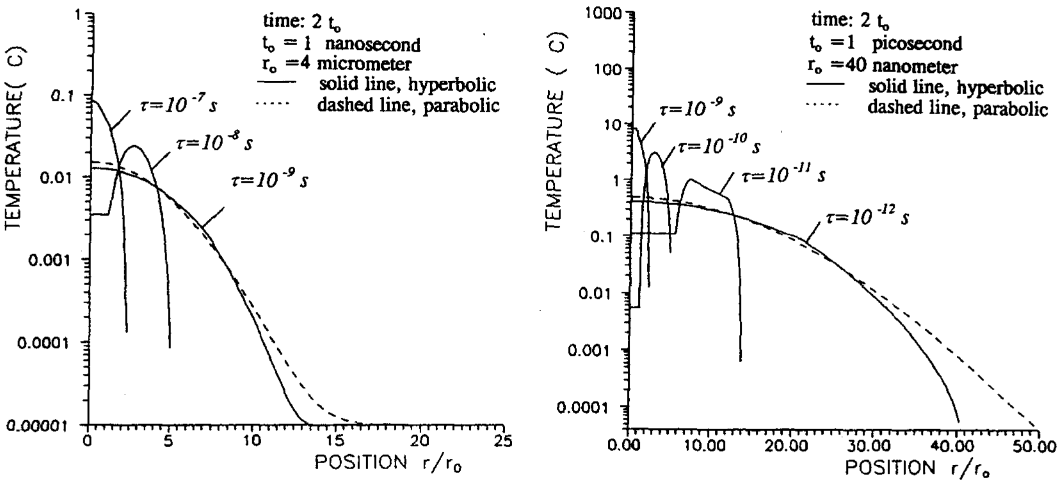
It is obvious that the hyperbolic and parabolic equations based on different mechanisms would give a different average junction temperature and eventually affect the evaluation of the thermal reliability.

The peak value in temperature profiles predicted by the hyperbolic equation can be as high as several times that by the parabolic. This higher peak temperature inevitably leads to the greater temperature gradient and the consequent higher thermal stroke, which will definitely speed up the failure rate of IC chips. It is well known that the thermal noise in the elements is critical for reliability of the working circuits. When the thermal noise in the medium is comparable with the working signal, an undesirable chaotic situation would occur. The probability of reversible failure related with thermal noise can greatly increase if a thermal wave appears, both for its higher peak temperature and its rapid oscillatory behavior.

A much sharper spatial temperature change can be expected based on heat wave phenomena in IC chips. This results in a great temperature difference on elements which would raise a problem of sign skew for analogue circuits and cause the time difference on data transmission for digital circuits.

In order to determine under what conditions the heat wave effect has to be considered in IC chips, a computational example will be given with typical parameters as: (1) the components size is 4 μm, as it is of the order of feature dimension for VLSI; (2) the frequency of electrical pulse is 10^8 Hz, which falls in to the range of 10^7–10^{12} Hz for IC; (3) the volumetrical energy generation, g_o, is 7.34×10^{13} W m^{-3} corresponding to total heat dissipation of about 3.7 W per chip, with assumed 10^5 elements being integrated on a chip; heat dissipation coming from 20% of the total elements and the pulse heating time is one-tenth of the shift period; (4) density $\rho = 2330$ kg m^{-3}, thermal capacity $C_p = 700$ J kg^{-1} K^{-1} and thermal conductivity $K = 100$ W m^{-1} K^{-1} for both doped region and pure region of medium.

The predicted temperature response to a heating pulse of 10^{-9} s in the solid with different relaxation time is represented in Fig. 3(a). It is clear that if the base material has a relaxation time longer than 10^{-9} s, hyperbolic and parabolic heat equations may give very different temperature profiles in the



(a) (b)

Fig. 3. Temperature profiles in the material with different relaxation times when a heating pulse of 10^{-9} s (a) or 10^{-12} s (b) is applied.

medium. The hyperbolic equation predicts a peaked temperature profile confined in a small region, in contrast, the parabolic one predicts a gentle temperature slope within a rather wide region. With the relaxation time shorter than 10^{-9} s, results from parabolic and hyperbolic equations are almost identical. Figure 3(b) shows that if the heat pulse period is as short as 10^{-12} s, corresponding to a working frequency of 10^{-11} Hz, heat wave effect is significant even in the medium with a relaxation time as short as 10^{-12} s, which falls in the range of relaxation time for most solids.

Noting that the critical value of relaxation time, which represents the lower limit for necessary consideration of heat wave effect, is in order of 10^{-9} s for a heating pulse period of 10^{-8} s [in Fig. 3(a)], and in order of 10^{-12} s for heating of 10^{-11} s [in Fig. 3(b)]; we can draw the conclusion that effect should be taken into consideration when the heating release time approaches to the relaxation time or more concretely, when the product of shift frequency and the relaxation time is of approximately the order of 0.1. This is very close to that given by Yuen and Lee [6], who investigated the transient heat conduction when an oscillatory heat flux was applied on the surface of a semi-infinite medium.

Some efforts have been put into the estimation or determination of relaxation time of various kinds of materials theoretically and experimentally [7–9]. The typical value of relaxation time widely used is in the order of 10^{-8}–10^{-10} s for gases at standard conditions and 10^{-10}–10^{-12} s for liquids and dielectric solids. For metals the typical value is between 10^{-12} and 10^{-14} s. Hence, according to our analysis assuming the relaxation time of silicon is the same order as that of dielectric solids, heat wave effects may be important at the upper limit of current density and working frequency of IC. We are already in a critical position to introduce wave effects in heat conduction analysis.

For most materials used in microelectronics, such as singular crystal silicon, a longer relaxation time could be expected because of the higher purity and perfect lattice structure with fewer vacancies and dislocations which strongly scatter phonons and lighten the wave behavior of heat propagation in silicon. In addition, a rather longer relaxation time can be expected if the chip is in cryogenic operation.

CONCLUSIONS

(1) The features of rapidly alternating current in IC chip and the perfect lattice structure of chip material may produce significant heat wave effects which cover a higher average junction temperature, a higher peak temperature, greater temperature difference between components and more rapid temperature change. These thermal effects are closely related to electrical failure mechanisms, such as electrical breakdown, thermal noise, sign skew and sign delay.

(2) A rough criterion is suggested, that is, if the product of shift frequency and the relaxation time of medium exceeds 0.1, it is necessary to take heat wave effects into consideration.

(3) Further theoretical and experimental work is still needed on the temperature wave behavior and the estimation of the relaxation time of chip materials.

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